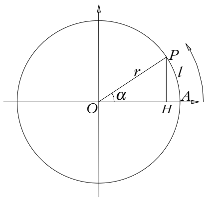
TRIGONOMETRIA

MISURA DI ANGOLI ED ARCHI



Circonf. generica: $\overline{OP} = r$
Circonf. goniometrica: $r = 1$
Periodicità: $f(\alpha + T) = f(\alpha)$
Radiante = angolo al centro relativo ad un arco lungo r
Grado = $1/90$ di angolo retto

Lunghezza arco: $\lambda = \dfrac{\overline{AP}}{\overline{OP}} = \dfrac{l}{r} \Rightarrow \boxed{\lambda = \alpha° \dfrac{\pi}{180°}}$

FUNZIONI TRIGONOMETRICHE

	Definizione	*Variazione*	*Periodo*	$\alpha=0$	$\pi/2$	π	$3\pi/2$	2π
$\sin\alpha$	$\overline{PH}/r$	$[-1,1]$	2π	0	1	0	-1	0
$\cos\alpha$	$\overline{OH}/r$	$[-1,1]$	2π	1	0	-1	0	1
$\text{tg}\,\alpha$	$\overline{AT}/r$	$\mathbb{R}$	π	0	$\nexists$	0	$\nexists$	0

$\alpha=$	$\pi/10=18°$	$\pi/6$	$\pi/4$	$\pi/3$	$2\pi/5=72°$
$\sin\alpha$	$(\sqrt{5}-1)/4$	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{10+2\sqrt{5}}/4$
$\cos\alpha$	$\sqrt{10+2\sqrt{5}}/4$	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	$(\sqrt{5}-1)/4$
$\text{tg}\,\alpha$	$\sqrt{1-\frac{2}{5}\sqrt{5}}$	$\sqrt{3}/3$	1	$\sqrt{3}$	$\sqrt{5+2\sqrt{5}}$

ARCHI ASSOCIATI

	$\pi/2\pm\alpha$	$\pi\pm\alpha$	$3\pi/2\pm\alpha$	$2k\pi\pm\alpha=\pm\alpha$
sin	$\cos\alpha$	$\mp\sin\alpha$	$-\cos\alpha$	$\pm\sin\alpha$
cos	$\mp\sin\alpha$	$-\cos\alpha$	$\pm\sin\alpha$	$\cos\alpha$
tg	$\mp 1/\text{tg}\,\alpha$	$\pm\text{tg}\,\alpha$	$\mp 1/\text{tg}\,\alpha$	$\pm\text{tg}\,\alpha$

IDENTITÀ FONDAMENTALI $\sin^2\alpha+\cos^2\alpha=1$, $\text{tg}\alpha=\sin\alpha/\cos\alpha$

IDENTITÀ NOTEVOLI

$\sin^2\alpha = \text{tg}^2\alpha/(1+\text{tg}^2\alpha)$ $\cos^2\alpha = 1/(1+\text{tg}^2\alpha)$

Addiz. e sottraz.	$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ $\text{tg}(\alpha\pm\beta) = (\text{tg}\alpha \pm \text{tg}\beta)/(1\mp\text{tg}\alpha\cdot\text{tg}\beta)$
Duplicaz.	$\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1-2\sin^2\alpha = 2\cos^2\alpha - 1$ $\text{tg}\,2\alpha = 2\,\text{tg}\,\alpha/(1-\text{tg}^2\alpha)$
Bisezione	$\sin^2\alpha/2=(1-\cos\alpha)/2$ $\cos^2\alpha/2=(1+\cos\alpha)/2$ $\tan^2\alpha/2 = (1-\cos\alpha)/(1+\cos\alpha)$

FORMULE DI PROSTAFERESI

$\cos\alpha+\cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$ $\cos\alpha-\cos\beta = -2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

$\sin\alpha\pm\sin\beta = 2\sin\dfrac{\alpha\pm\beta}{2}\cos\dfrac{\alpha\mp\beta}{2}$ $\text{tg}\alpha\pm\text{tg}\beta = \sin(\alpha\pm\beta)/(\cos\alpha\cos\beta)$

FORMULE DI WERNER

$$\sin\alpha\cdot\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

$$\cos\alpha\cdot\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$$

$$\sin\alpha\cdot\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

FORMULE RAZIONALI

$t = \text{tg}\alpha/2 \Rightarrow \sin\alpha = 2t/(1+t^2), \cos\alpha = (1-t^2)/(1+t^2)$

EQUAZIONI FONDAMENTALI

$\sin x = s,\ \alpha = \arcsin s \quad \Rightarrow x = (-1)^k\alpha + 2k\pi$
$\sin x = 0 \quad \Rightarrow x = k\pi$
$\cos x = c,\ \alpha = \arccos c \quad \Rightarrow x = \pm\alpha + 2k\pi$
$\cos x = 0 \quad \Rightarrow x = \pi/2 + k\pi$
$\text{tg}\,x = t,\ \alpha = \text{arctg}\,t \quad \Rightarrow x = \alpha + k\pi$
$\text{tg}\,x = 0 \quad \Rightarrow x = k\pi$

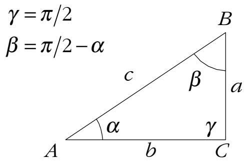
$k \in \mathbb{Z}$

DISEQUAZIONI

Si risolvono per via grafica, una volta individuate le soluzioni dell'eq. corrispondente.

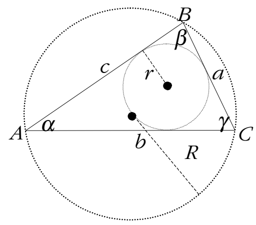
TRIANGOLO RETTANGOLO

$\gamma = \pi/2$
$\beta = \pi/2 - \alpha$



$a = c\cos\beta = c\sin\alpha = b\,\text{tg}\,\alpha$
$b = c\cos\alpha = c\sin\beta = a\,\text{tg}\,\beta$
$c = a/\sin\alpha = b/\sin\beta =$
$= b/\cos\alpha = a/\cos\beta$

TRIANGOLO QUALUNQUE



$\boxed{\alpha+\beta+\gamma=\pi}$

T. proiezioni:
$a = c\cos\beta + b\cos\gamma$
$b = a\cos\gamma + c\cos\alpha$
$c = b\cos\alpha + a\cos\beta$

T. Carnot:
$a^2 = b^2 + c^2 - 2bc\cos\alpha$

T. Nepero:
$$\frac{a+b}{a-b} = \frac{\text{tg}\frac{\alpha+\beta}{2}}{\text{tg}\frac{\alpha-\beta}{2}}$$

T. seni:
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$$

AREA TRIANGOLO

$S = \dfrac{1}{2}bc\sin\alpha = \dfrac{1}{2}a^2\dfrac{\sin\beta\sin\gamma}{\sin\alpha}$

$S = \sqrt{p\,(p-a)(p-b)(p-c)}$ *(formula di Erone)*

$S = r\,p = \dfrac{abc}{4R} = \dfrac{(p-a)(p-b)(p-c)}{r} = 2R^2\sin\alpha\sin\beta\sin\gamma$

$S = \dfrac{r^2}{\text{tg}\alpha/2\cdot\text{tg}\beta/2\cdot\text{tg}\gamma/2} = p^2\,\text{tg}\alpha/2\cdot\text{tg}\beta/2\cdot\text{tg}\gamma/2$

Relativamente al lato a:

Altezza	*Mediana*	*Bisettrice*
$h_a = 2S/a$	$m_a = \frac{1}{2}\sqrt{2(b^2+c^2)-a^2}$	$k_\alpha = \dfrac{2bc}{b+c}\cos\dfrac{\alpha}{2}$

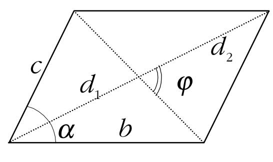
FORMULE DI BRIGGS

$\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{(p-b)(p-c)}{bc}}$, $\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{p(p-a)}{bc}}$, $\text{tg}\dfrac{\alpha}{2} = \sqrt{\dfrac{(p-b)(p-c)}{p(p-a)}}$

Circonf. inscritta: $r = S/p = (p-a)\,\text{tg}\alpha/2$

Circonf. circoscritta: $R = \dfrac{abc}{4S} = \dfrac{a}{2\sin\alpha}$

AREA QUADRILATERO



Area parallelogr.: $S = bc\sin\alpha$

Area quadrilatero: $S = \frac{1}{2}d_1 d_2\sin\varphi$