

# INTEGRALI INDEFINITI NOTEVOLI

N.B. È stata omessa la costante arbitraria.

## IMMEDIATI

$\int dx$	$x$	$\int x dx$	$\frac{x^2}{2}$
$\int x^n dx$ ( $n \neq -1$ )	$\frac{x^{n+1}}{n+1}$	$\int x^{-n} dx$ ( $n \neq 1$ )	$-\frac{1}{(n-1)x^{n-1}}$
$\int \frac{1}{x} dx$	$\ln x $	$\int \frac{1}{x^2} dx$	$-\frac{1}{x}$
$\int \sqrt{x} dx$	$\frac{2}{3}\sqrt{x^3}$	$\int \frac{1}{\sqrt{x}} dx$	$2\sqrt{x}$
$\int \sin x dx$	$-\cos x$	$\int \cos x dx$	$\sin x$
$\int \sin ax dx$	$-\frac{1}{a}\cos ax$	$\int \cos ax dx$	$\frac{1}{a}\sin ax$
$\int \sin \frac{x}{a} dx$	$-a\cos \frac{x}{a}$	$\int \cos \frac{x}{a} dx$	$a\sin \frac{x}{a}$
$\int \frac{1}{\sin x} dx$	$\ln\left \operatorname{tg} \frac{x}{2}\right  = \ln \operatorname{cosec} x - \operatorname{ctg} x $	$\int \frac{1}{\cos x} dx$	$\ln\left \operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{4}\right)\right  = \ln \sec x + \operatorname{tg} x $
$\int \operatorname{tg} x dx$	$-\ln \cos x $	$\int \frac{1}{\operatorname{tg} x} dx$	$\ln \sin x $
$\int \frac{1}{\sin^2 x} dx$	$-\frac{1}{\operatorname{tg} x}$	$\int \frac{1}{\cos^2 x} dx$	$\operatorname{tg} x$
$\int \operatorname{arctg} x dx$	$x \operatorname{arctg} x - \frac{1}{2}\ln(1+x^2)$	$\int \operatorname{arcsen} x dx$	$\sqrt{1-x^2} + x \operatorname{arcsen} x$
$\int e^x dx$	$e^x$	$\int a^x dx$	$\frac{1}{\ln a} a^x$
$\int \ln x dx$	$x(\ln x - 1)$	$\int x^n \ln x dx$	$\frac{x^{n+1}}{n+1} \left(\ln x - \frac{1}{n+1}\right)$
$\int \frac{1}{\sqrt{1-x^2}} dx$	$\operatorname{arcsen} x$	$\int \frac{1}{1+x^2} dx$	$\operatorname{arctg} x = \operatorname{arcsen} \frac{x}{\sqrt{1+x^2}}$
$\int \frac{x}{\sqrt{a-x^2}} dx$	$-\sqrt{a-x^2}$ ( $a > 0$ )	$\int \frac{x}{\sqrt{a+x^2}} dx$	$\sqrt{a+x^2}$
$\int \sinh x dx$	$\cosh x$	$\int \cosh x dx$	$\sinh x$
$\int \frac{1}{\sinh^2 x} dx$	$-\frac{1}{\operatorname{tgh} x}$	$\int \frac{1}{\cosh^2 x} dx$	$\operatorname{tanh} x$

## FUNZIONE LINEARE

$$L = ax + b$$

$\int L^n dx$	$\frac{L^{n+1}}{a(n+1)}$ ( $n \neq -1$ )
$\int \frac{1}{L} dx$	$\frac{1}{a} \ln L $
$\int \frac{1}{L^n} dx$	$-\frac{1}{a(n-1)L^{n-1}}$ ( $n \neq 1$ )
$\int \frac{x}{L} dx$	$\frac{1}{a^2} [L - b \ln L ]$
$\int \frac{x}{L^n} dx$	$\frac{1}{a^2} \left[ -\frac{1}{(n-2)L^{n-2}} + \frac{b}{(n-1)L^{n-1}} \right] = \frac{L^{1-n}(ax - b - anx)}{a^2(n^2 - 3n + 2)}$
$\int \frac{x^2}{L} dx$	$\frac{x(ax - 2b)}{2a^2} + \frac{b^2 \ln L }{a^3}$
$\int \frac{x^2}{L^n} dx$	$\frac{1}{a^3} \left[ -\frac{1}{(n-3)L^{n-3}} + \frac{2b}{(n-2)L^{n-2}} - \frac{b^2}{(n-1)L^{n-1}} \right]$

$\int \frac{x^3}{L} dx$	$\frac{b^2x}{a^3} - \frac{bx^2}{2a^2} + \frac{x^3}{3a} - \frac{b^3}{a^4} \ln L $
$\int \frac{1}{xL} dx$	$\frac{\ln x  - \ln L }{b}$
$\int \frac{1}{xL^2} dx$	$\frac{1}{bL} + \frac{\ln x  - \ln L }{b^2}$
$\int \frac{1}{xL^3} dx$	$\frac{1}{2bL^2} + \frac{1}{b^2L} + \frac{\ln x  - \ln L }{b^3}$
$\int \frac{1}{x^2L} dx$	$-\frac{b+ax \ln x }{b^2x} + \frac{a \ln L }{b^2}$
$\int \frac{1}{x^2L^2} dx$	$-\frac{1}{b^2x} - \frac{a}{b^2L} - 2a \frac{\ln x  - \ln L }{b^3}$
$\int \frac{1}{x^2L^3} dx$	$-\frac{1}{b^3x} - \frac{a}{2b^2L^2} - \frac{2a}{b^3L} - 3a \frac{\ln x  - \ln L }{b^4}$

$$A = a + x, B = b + x$$

$\int \frac{1}{AB} dx$	$-\frac{\ln A  - \ln B }{a-b}$
$\int \frac{x}{A^2B} dx$	$\frac{1}{(a-b)A} - \frac{\ln A  - \ln B }{(a-b)^2}$
$\int \frac{1}{AB^2} dx$	$-\frac{1}{(a-b)B} + \frac{\ln A  - \ln B }{(a-b)^2}$
$\int \frac{1}{A^2B^2} dx$	$-\frac{1}{(a-b)^2A} - \frac{1}{(a-b)^2B} + 2 \frac{\ln A  - \ln B }{(a-b)^3}$

## FUNZIONE QUADRATICA

	$Q = x^2 + a^2$	$Q = a^2 - x^2$
$\int \frac{1}{Q} dx$	$\frac{1}{a} \operatorname{arctg} \frac{x}{a}$	$\frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  = \frac{1}{a} \operatorname{arctgh} \frac{x}{a}$
$\int \frac{1}{Q^2} dx$	$\frac{x}{2a^2Q} + \frac{1}{2a^3} \operatorname{arctg} \frac{x}{a}$	$\frac{x}{2a^2Q} + \frac{1}{2a^3} \operatorname{arctgh} \frac{x}{a}$
$\int \frac{x}{Q} dx$	$\frac{\ln Q }{2}$	$-\frac{\ln Q }{2}$
$\int \frac{x}{Q^2} dx$	$-\frac{1}{2Q}$	$\frac{1}{2Q}$
$\int \frac{x}{Q^3} dx$	$-\frac{1}{4Q^2}$	$\frac{1}{4Q^2}$
$\int \frac{x}{Q^n} dx$	$\frac{Q^{1-n}}{2(1-n)}$	$-\frac{Q^{1-n}}{2(1-n)}$
$\int \frac{x^2}{Q} dx$	$x - a \operatorname{arctg} \frac{x}{a}$	$-x + a \operatorname{arctgh} \frac{x}{a}$
$\int \frac{x^2}{Q^2} dx$	$-\frac{x}{2Q} + \frac{1}{2a} \operatorname{arctg} \frac{x}{a}$	$\frac{x}{2Q} - \frac{1}{2a} \operatorname{arctgh} \frac{x}{a}$
$\int \frac{x^3}{Q^2} dx$	$\frac{a^2 + \ln Q }{2Q}$	$\frac{a^2}{2Q} + \frac{\ln Q }{2}$
$\int \frac{1}{xQ} dx$	$\frac{2 \ln x  - \ln Q }{2a^2}$	$\frac{2 \ln x  - \ln Q }{2a^2}$
$\int \frac{1}{xQ^2} dx$	$\frac{1}{2a^2Q} + \frac{2 \ln x  - \ln Q }{2a^4}$	$\frac{1}{2a^2Q} + \frac{2 \ln x  - \ln Q }{2a^4}$
$\int \frac{1}{x^2Q} dx$	$-\frac{1}{a^2x} - \frac{1}{a^2} \operatorname{arctg} \frac{x}{a}$	$-\frac{1}{a^2x} + \frac{1}{a^2} \operatorname{arctgh} \frac{x}{a}$
$\int \frac{1}{x^2Q^2} dx$	$-\frac{1}{a^4x} - \frac{x}{2a^4Q} - \frac{3}{2a^5} \operatorname{arctg} \frac{x}{a}$	$-\frac{1}{a^4x} + \frac{x}{2a^4Q} + \frac{3}{2a^5} \operatorname{arctgh} \frac{x}{a}$

$$P = ax^2 + bx + c, \quad \Delta = b^2 - 4ac$$

$\int \frac{1}{P} dx$	$-\frac{2}{\sqrt{\Delta}} \operatorname{arctgh} \frac{b+2ax}{\sqrt{\Delta}}$
$\int \frac{1}{P^2} dx$	$-\frac{b+2ax}{\Delta P} + \frac{4a}{\sqrt{\Delta^3}} \operatorname{arctgh} \frac{b+2ax}{\sqrt{\Delta}}$
$\int \frac{x}{P} dx$	$\frac{b}{a\sqrt{\Delta}} \operatorname{arctgh} \frac{b+2ax}{\sqrt{\Delta}} + \frac{\ln P }{2a}$
$\int \frac{x}{P^2} dx$	$\frac{2c+bx}{\Delta P} - \frac{2b}{\sqrt{\Delta^3}} \operatorname{arctgh} \frac{b+2ax}{\sqrt{\Delta}}$
$\int \frac{1}{xP} dx$	$\frac{b}{c\sqrt{\Delta}} \operatorname{arctgh} \frac{b+2ax}{\sqrt{\Delta}} + \frac{2\ln x  - \ln P }{2c}$
$\int \frac{1}{xP^2} dx$	$\frac{bP-2ac}{c\Delta P} + \frac{b^2-6abc}{c^2\sqrt{\Delta^3}} \operatorname{arctgh} \frac{b+2ax}{\sqrt{\Delta}} + \frac{2\ln x  - \ln P }{2c^2}$

### FUNZIONI IRRAZIONALI

	$F = \sqrt{x^2 - a^2} \quad (x^2 > a^2)$	$F = \sqrt{a^2 - x^2} \quad (x^2 < a^2)$
$\int \frac{1}{F} dx$	$\ln x+F $	$\arcsin \frac{x}{a} = \operatorname{arctg} \frac{x}{F}$
$\int \frac{1}{xF} dx$	$-\frac{1}{a} \operatorname{arctg} \frac{a}{F}$	$\frac{1}{a} \ln \left  \frac{x}{a(a+F)} \right $
$\int \frac{1}{xF^3} dx$	$-\frac{1}{a^2 F} + \frac{1}{a^3} \operatorname{arctg} \frac{a}{F}$	$\frac{1}{a^2 F} + \frac{1}{a^3} \ln \left  \frac{x}{a^3(a+F)} \right $
$\int \frac{1}{x^2 F} dx$	$\frac{F}{a^2 x}$	$-\frac{F}{a^2 x}$
$\int F dx$	$\frac{1}{2}(xF - a^2 \ln x+F )$	$\frac{1}{2} \left( xF + a^2 \arcsin \frac{x}{a} \right)$
$\int F^3 dx$	$\frac{F}{4}(x^3 - \frac{5}{2}a^2 x) + \frac{3}{8}a^4 \ln x+F $	$\frac{F}{4}(\frac{5}{2}a^2 x - x^3) + \frac{3}{8}a^4 \arcsin \frac{x}{a}$
$\int xF^3 dx$	$\frac{1}{5}F^5$	$-\frac{1}{5}F^5$
$\int x^2 F dx$	$\frac{F}{4} \left( x^3 - \frac{a^2 x}{2} \right) - \frac{a^4}{8} \ln x+F $	$\frac{F}{4} (x^3 - \frac{1}{2}a^2 x) + \frac{1}{8}a^4 \arcsin \frac{x}{a}$
$\int \frac{F}{x} dx$	$F + a \operatorname{arctg} \frac{a}{F}$	$F + a \ln \left  \frac{x}{a(a+F)} \right $
$\int \frac{F}{x^2} dx$	$-\frac{F}{x} + \ln x+F $	$-\frac{F}{x} - \arcsin \frac{x}{a}$
$\int \frac{F^3}{x} dx$	$\frac{x^2 - 4a^2}{3} F - a^3 \operatorname{arctg} \frac{a}{F}$	$\frac{4a^2 - x^2}{3} F + a^3 \ln \left  \frac{x}{a(a+F)} \right $

$$G = \sqrt{x^2 + a^2}$$

$\int \frac{1}{G} dx$	$\ln x+G $
$\int \frac{1}{G^3} dx$	$\frac{x}{a^2 G}$
$\int \frac{x^3}{G} dx$	$\frac{x^2 - 2a^2}{3} G$
$\int \frac{1}{xG} dx$	$\frac{1}{a} \ln \left  \frac{x}{a(a+G)} \right $
$\int \frac{1}{xG^3} dx$	$\frac{1}{a^2 G} + \frac{1}{a^3} \ln \left  \frac{x}{a^3(a+G)} \right $
$\int G dx$	$\frac{1}{2}(xG + a^2 \ln x+G )$
$\int G^3 dx$	$G \left( \frac{5a^2 x}{8} + \frac{x^3}{4} \right) + \frac{3a^4}{8} \ln x+G $
$\int xG^3 dx$	$\frac{1}{5}G^5$

$\int \frac{G}{x} dx$	$G + a \ln \left  \frac{x}{a(a+G)} \right $
$\int \frac{G^3}{x} dx$	$\frac{x^2 + 4a^2}{3} G + a^3 \ln \left  \frac{x}{a(a+G)} \right $
$\int \frac{G}{x^2} dx$	$-\frac{G}{x} + \ln x+G $

### FUNZIONI TRIGONOMETRICHE

	$F = \sin x$	$F = \cos x$
$\int xF dx$	$-x \cos x + \sin x$	$\cos x + x \sin x$
$\int x^2 F dx$	$(2 - x^2) \cos x + 2x \sin x$	$2x \cos x + (x^2 - 2) \sin x$
$\int F^2 dx$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\frac{x}{2} + \frac{\sin 2x}{4}$
$\int xF^2 dx$	$\frac{x^2}{4} - \frac{\cos 2x}{8} - \frac{x \sin 2x}{4}$	$\frac{x^2}{4} + \frac{\cos 2x}{8} + \frac{x \sin 2x}{4}$
$\int x^2 F^2 dx$	$\frac{x^3}{6} - \frac{x \cos 2x}{4} + \frac{(1-2x^2) \sin 2x}{8}$	$\frac{x^3}{6} + \frac{x \cos 2x}{4} - \frac{(1-2x^2) \sin 2x}{8}$
$\int F^3 dx$	$-\frac{3}{4} \cos x + \frac{\cos 3x}{12}$	$\frac{3}{4} \sin x + \frac{\sin 3x}{12}$
$\int xF^3 dx$	$-\frac{3x \cos x}{4} + \frac{x \cos 3x}{12} + \frac{3 \sin x}{4} - \frac{\sin 3x}{36}$	$\frac{3 \cos x}{4} + \frac{\cos 3x}{36} + \frac{3x \sin x}{4} + \frac{x \sin 3x}{12}$
$\int \frac{1}{1+F} dx$	$\frac{2 \operatorname{sen} \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}}$	$\operatorname{tg} \frac{x}{2}$
$\int \frac{1}{1-F} dx$	$\frac{2 \operatorname{sen} \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}}$	$-\operatorname{cotg} \frac{x}{2}$
$\int \frac{F}{1+F} dx$	$\frac{x \cos \frac{x}{2} + (x-2) \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}}$	$x - \operatorname{tg} \frac{x}{2}$
$\int \frac{F}{1-F} dx$	$\frac{-x \cos \frac{x}{2} + (x+2) \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}}$	$-x - \operatorname{cotg} \frac{x}{2}$