Limiti notevoli

Trigonometriche

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \lim_{x\to 0}\frac{\sin x^n}{x^n}=1 \qquad \lim_{x\to\infty}\frac{\sin x}{x}=0$$

$$\lim_{x\to 0}\frac{\sin \alpha x}{x}=\alpha \qquad \lim_{x\to 0}\frac{\sin \alpha x}{\sin \beta x}=\frac{\alpha}{\beta} \qquad \nexists \lim_{x\to\infty}\sin x$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2} \qquad \lim_{x\to 0}\frac{1-\cos x}{x}=0 \qquad \nexists \lim_{x\to\infty}\cos x$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x}=1 \qquad \lim_{x\to 0}\frac{\operatorname{tg} \alpha x}{x}=\alpha \qquad \lim_{x\to 0}\frac{\operatorname{tg} \alpha x}{\operatorname{tg} \beta x}=\frac{\alpha}{\beta}$$

Esponenziale - Logaritmo - Potenza

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e \qquad \lim_{x\to 0}(1+x)^{1/x}=e \qquad \lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x=\frac{1}{e}$$

$$\lim_{x\to+\infty}\frac{e^x}{x^\alpha}=+\infty \qquad \lim_{x\to\infty}\left(1+\frac{\alpha}{x}\right)^{\beta x}=e^{\alpha\beta} \qquad \lim_{x\to 0}\frac{(1+x)^\alpha-1}{\alpha x}=1$$

$$\lim_{x\to+\infty}\frac{x^\alpha}{e^x}=0 \qquad \lim_{x\to 0}\frac{e^x-1}{x}=1 \qquad \lim_{x\to 0}\frac{a^x-1}{x}=\log a$$

$$\lim_{x\to+\infty}\frac{\log x}{x^\alpha}=0 \qquad \lim_{x\to 0^+}x^\alpha\log x=0 \qquad \lim_{x\to 0}\frac{\log(1+x)}{x}=1$$

$$\lim_{x\to 0}x^x=1 \qquad \lim_{x\to+\infty}x^{1/x}=1 \qquad \lim_{x\to+\infty}x^\alpha\log x=+\infty$$

$$\lim_{x\to\infty}\frac{1}{1+a^x}=\begin{cases}1 & \text{se } a<1\\ 0 & \text{se } a>1\end{cases} \qquad \lim_{x\to+\infty}(x-a\log x)=+\infty$$

Infinitesimi equivalenti per $x\to 0$

$\sin\alpha \sim \alpha$	$\operatorname{tg}\alpha \sim \alpha$	$1-\cos\alpha \sim \alpha^2/2$
$\arcsin\alpha \sim \alpha$	$\operatorname{arctg}\alpha \sim \alpha$	$\log(1+\alpha)\sim\alpha$
$e^\alpha-1\sim\alpha$	$a^\alpha-1\sim\alpha\log a$	$(1+\alpha)^k-1\sim k\alpha$

Derivazione notevoli

Regole di Derivazione

- *Combinazione lineare*
  $$D[a_1f_1(x)+\ldots+a_nf_n(x)]=a_1f_1'(x)+\ldots+a_nf_n'(x)$$
- *Prodotto* $\quad D[f(x)\cdot g(x)]=f'(x)\cdot g(x)+f(x)\cdot \dot g'(x)$
- *Potenza* $\quad D\,f^\alpha(x)=\alpha\, f^{\alpha-1}(x)\cdot f'(x)$
- *Reciproca* $\quad D\frac{1}{g(x)}=-\frac{g'(x)}{g^2(x)}$
- *Rapporto* $\quad D\frac{f(x)}{g(x)}=\frac{f'(x)\cdot g(x)-f(x)\cdot \dot g'(x)}{g^2(x)}$
- *Composta* $\quad D\,g(f(x))=g'(f(x))\cdot f'(x)$
- *Inversa* $\quad D\,f^{-1}(y)=\frac{1}{f'(x)} \Leftrightarrow f'(x)=\frac{1}{D\,f^{-1}(y)}$
- *Esponenziale (derivata logaritmica)*
  $$D[f(x)]^{g(x)}=[f(x)]^{g(x)}\left(g(x)\frac{f'(x)}{f(x)}+g'(x)\cdot\log f(x)\right)$$

Derivate elementari

$Dk=0$	$D\,a^x=a^x\log a$
$D\,x=1$	$D\,e^x=e^x$
$D(ax+b)=a$	$D\log x=\frac{1}{x}$
$D\,x^\alpha=\alpha\,x^{\alpha-1}$	$D\log_a x=\frac{1}{x\log a}$
$D\frac{1}{x}=-\frac{1}{x^2}$	$D\log_x a=-\frac{\log a}{x\log^2 a}$
$D\sqrt{x}=\frac{1}{2\sqrt{x}}$	$D\,x^x=x^x(1+\log x)$
$D\sqrt[n]{x^m}=\frac{m}{n}\sqrt[n]{x^{m-n}}$	$D\,x(\log x-1)=\log x$
$D\sin x=\cos x$	$D\arcsin x=\frac{1}{\sqrt{1-x^2}}$
$D\cos x=-\sin x$	$D\arccos x=-\frac{1}{\sqrt{1-x^2}}$
$D\operatorname{tg}x=\frac{1}{\cos^2 x}=1+\operatorname{tg}^2 x$	$D\operatorname{arctg}x=\frac{1}{1+x^2}$
$D(\operatorname{tg}x-x)=\operatorname{tg}^2 x$	$D\arccos\frac{-x}{\sqrt{1+x^2}}=\frac{1}{1+x^2}$
$D\frac{1-\cos x}{1+\cos x}=\frac{\operatorname{tg}\frac{x}{2}}{\cos^2\frac{x}{2}}$	$D\frac{1-\operatorname{tg}x}{1+\operatorname{tg}x}=\frac{2}{1+\sin 2x}$

Derivate n-sime fondamentali

$$D^n x^\alpha=\alpha(\alpha-1)\ldots(\alpha-n+1)x^{\alpha-n} \qquad D^n x^n=n!$$

$$D^n(a_0+a_1x+\ldots+a_nx^n)=a_n n! \qquad D^n a^x=a^x\log^n a$$

$$D^n\sin x=\sin\left(x+n\tfrac{\pi}{2}\right) \qquad D^n\sin\alpha x=\alpha^n\sin\left(\alpha x+n\tfrac{\pi}{2}\right)$$

$$D^n\cos x=\cos\left(x+n\tfrac{\pi}{2}\right) \qquad D^n\cos\alpha x=\alpha^n\cos\left(\alpha x+n\tfrac{\pi}{2}\right)$$

Teoremi di De L'Hôpital

Forma Indet.	Trasformazione	
$\frac{0}{0},\frac{\infty}{\infty}$		$\lim_{x\to x_0}\frac{f(x)}{g(x)}=\lim_{x\to x_0}\frac{f'(x)}{g'(x)}=\ldots=\lim_{x\to x_0}\frac{f^{(n)}(x)}{g^{(n)}(x)}$ ∃:stop
$0\cdot\infty$	$\frac{0}{0}$ o $\frac{\infty}{\infty}$	$f(x)\cdot g(x)=\frac{f(x)}{\frac{1}{g(x)}}=\frac{g(x)}{\frac{1}{f(x)}}$
$+\infty-\infty$	$\frac{\infty}{\infty}$	$f(x)-g(x)=f(x)\left(1-\frac{g(x)}{f(x)}\right)$
$0^0,1^\infty,\infty^0,0^\infty$	$0\cdot\infty$	$[f(x)]^{g(x)}=e^{g(x)\log f(x)}$
$\log_0 0,\ \log_0\infty,\ \log_\infty 0,\ \log_\infty\infty$	$\frac{\infty}{\infty}$	$\log_{f(x)}g(x)=\frac{\log g(x)}{\log f(x)}$
$\log_1 1$	$\frac{0}{0}$	