Numeri Complessi

Unità immaginaria i: $i^2 = -1$

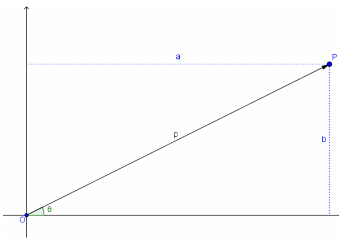
Numeri complessi: $\mathbb{C} = \left\{ z = a + ib \,\middle|\, a, b \in \mathbb{R};\ i^2 = -1 \right\}$

Numeri immaginari: $ib \mid b \in \mathbb{R};\ i^2 = -1$

Potenze di *i*

$i^0 = 1$	$i^1 = i$	$i^2 = -1$	$i^3 = -i$
$i^4 = 1$	$i^5 = i$	$i^6 = -1$	…
$i^{4k} = 1$	$i^{4k+1} = i$	$i^{4k+2} = -1$	$i^{4k+3} = -i$

Coordinate



$$\begin{cases} a = \rho \cos \vartheta \\ b = \rho \sin \vartheta \end{cases} \Updownarrow \begin{cases} \rho = \sqrt{a^2 + b^2} \\ \mathrm{tg}\,\vartheta = b/a \end{cases}$$

$a = \mathrm{Re}(z)$: parte reale $\rho = |z|$: modulo

$b = \mathrm{Im}(z)$: parte immaginaria ϑ: argomento

Forme

$$z = \underbrace{a + ib}_{\text{Algebrica}} = \underbrace{\rho(\cos\vartheta + i \sin\vartheta)}_{\text{Trigonometrica}} = \underbrace{\rho e^{i\vartheta}}_{\text{Esponenziale}}$$

Operazioni

coniugato $\bar{z} = a - ib = \rho(\cos(-\vartheta) + i\sin(-\vartheta))$

opposto $-z = -a - ib = \rho(\cos(\vartheta + \pi) + i \sin(\vartheta + \pi))$

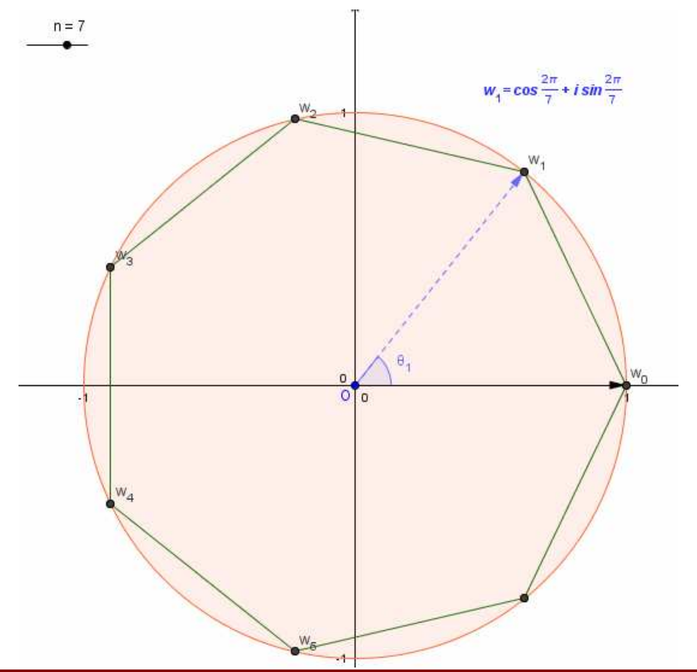
Proprietà del coniugato

$z + \bar{z} = 2a \in \mathbb{R}$ $z \cdot \bar{z} = a^2 + b^2 = |z|^2 = \rho^2 > 0$

	Somma
A	$(a + ib) + (c + id) = (a + c) + i(b + d)$
	Prodotto
A	$(a + ib)\cdot(c + id) = (ac - bd) + i(bc + ad)$
T	$\rho_1(\cos\vartheta_1 + i \sin\vartheta_1)\cdot\rho_2(\cos\vartheta_2 + i\sin\vartheta_2) = \rho_1\rho_2(\cos(\vartheta_1 + \vartheta_2) + i\sin(\vartheta_1+\vartheta_2))$
E	$\rho_1 e^{i\vartheta_1}\cdot\rho_2 e^{i\vartheta_2} = \rho_1\rho_2 e^{i(\vartheta_1+\vartheta_2)}$
	Reciproco
A	$\frac{1}{c + id} = \frac{c - id}{c^2 + d^2} = \frac{c}{c^2+d^2} - i\frac{d}{c^2+d^2}$
T	$\frac{1}{\rho(\cos\vartheta + i\sin\vartheta)} = \frac{1}{\rho}(\cos(-\vartheta) + i\sin(-\vartheta))$
E	$\frac{1}{\rho e^{i\vartheta}} = \frac{1}{\rho}e^{-i\vartheta}$

	Rapporto
A	$\frac{a + ib}{c + id} = \frac{ac + bd}{c^2 + d^2} + i\frac{bc - ad}{c^2 + d^2}$
T	$\frac{\rho_1(\cos\vartheta_1 + i\sin\vartheta_1)}{\rho_2(\cos\vartheta_2 + i\sin\vartheta_2)} = \frac{\rho_1}{\rho_2}(\cos(\vartheta_1 - \vartheta_2) + i\sin(\vartheta_1 - \vartheta_2))$
E	$\frac{\rho_1 e^{i\vartheta_1}}{\rho_2 e^{i\vartheta_2}} = \frac{\rho_1}{\rho_2}e^{i(\vartheta_1 - \vartheta_2)}$
	Potenza
A	$(a + ib)^n$ = potenza di binomio (Tartaglia)
T	$[\rho(\cos\vartheta + i\sin\vartheta)]^n = \rho^n(\cos n\vartheta + i \sin n\vartheta)$ De Moivre
E	$(\rho e^{i\vartheta})^n = \rho^n e^{in\vartheta}$
	Radici
T	$\sqrt[n]{z^*} = \sqrt[n]{\rho}\left(\cos\frac{\vartheta + 2k\pi}{n} + i\sin\frac{\vartheta+2k\pi}{n}\right)$ $k = 0, \ldots, n-1$
E	$\sqrt[n]{z^*} = \sqrt[n]{\rho}\, e^{i\frac{\vartheta+2k\pi}{n}}$ $k = 0, \ldots, n-1$

Radici dell'unità



Formula di Eulero: $e^{i\pi} + 1 = 0$

Funz. circolari: $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$, $\cos x = \frac{e^{ix} + e^{-ix}}{2}$

Funz. iperboliche: $\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$

$\sinh(ix) = \sin x$, $\cosh(ix) = \cos x$