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Prove di CLIL //

Chaotic Butterflies

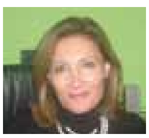
Nicola Chiriano / Caterina Oliverio

Docenti al Liceo scientifico “Siciliani” di Catanzaro

con Chiara Scarpino (5G), Francesco Bitonti (5F) e Domenico Soluri (5F)

[NICOLA CHIRIANO]

Nicola Chiriano è docente di Matematica e Fisica al Liceo scientifico “Siciliani” di Catanzaro. Si occupa di didattica e ICT. È formatore in diversi corsi per docenti e studenti di vari ordini di scuola. Ha all’attivo varie collaborazioni con Ansas (e-tutor nei corsi Pon Tec) e Invalsi (piani di formazione Ocse-Pisa e SNV). Su A&B ha già proposto un percorso tra Musica e Matematica.



[CATERINA OLIVERIO]

Caterina Oliverio, laureata in Lingue e Letterature straniere presso l'Università di Bari, è docente di Lingua e Letteratura inglese nelle scuole superiori dal 1992. Dal 2000 insegna al Liceo scientifico "Siciliani" di Catanzaro, dove si occupa di English for Specific Purposes. Collabora a contratto con la Facoltà di Medicina e Chirurgia dell'Università "Magna Graecia" di Catanzaro.

[TOPICS NEEDED]

- Definitions of a function and of sequences
- Simple types of functions, periodicity
- Dependence on a parameter
- Concept of limit

[OBJECTIVES]

- Being able to describe the Butterfly Effect
- Understanding what is the Orbit of a map
- Describing the properties of a simple Logistic map
- Listing daily applications of Chaos Theory

["THE BUTTERFLY EFFECT"]

This term was first used in a novel written in 1952 by **Ray Bradbury**, "*A Sound of Thunder*": the plot is thrilling and deals with travels in time organized for tourists by means of a time-machine. One of the travellers in the future experiences the catastrophic consequences of an apparently banal incident: he has stomped a butterfly in the past.

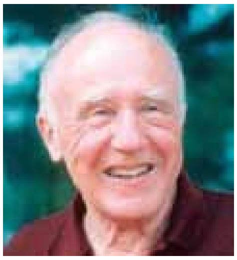


Fig. 1 - Edward Norton Lorenz (1917 - 2008)

At a Conference in 1972, the American physicist **Edward N. Lorenz** assumed that

"the flap of a butterfly wing in Brazil can set off a tornado in Texas"

In other words, in some situations even a tiny difference in initial conditions can result, in the long run, in unpredictable variations. The result is a "**chaotic**" system of this kind, like the flow of a waterfall, the drip of a tap, the movements of clouds, the dynamic of fluids in general, including human heart pumping, but also earthquakes, financial flows and so on.

All disciplines offer amazing applications and confirmations on **Chaos Theory.** It has been shown that Chaos is a natural behavior of phenomena and life itself is made possible just because Chaos exists.

Chaos Theory is often referred to in novels or movies, such as Spielberg's famous "*Jurassic Park*" or as the less known "*Butterfly Effect*", in which every detail amended in the past by the leading actor permanently alters his future and the one of the people he cares about.

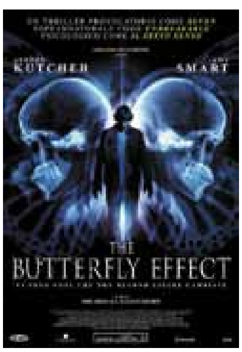


Fig. 2 – "Jurassic Park" (USA 1993)

Fig. 3 - "The Butterfly Effect" (USA 2004)

[DYNAMIC SYSTEMS]

Despite of its unpredictable behavior, Chaos has an amazingly simple formulation, so it is **deterministic** in principle. A dynamical system is in fact a mathematical model, which is a **difference equation** that describes how the state of a (physical, biological, economic...) system evolves over time: the following value is the transformation of the former one via a function **f:**

$$x_{n+1} = f(x_n) \ .$$

We call **orbit** the sequence of values $x_0 \longrightarrow x_1 \longrightarrow x_2 \longrightarrow \dots \longrightarrow x_n \longrightarrow x_{n+1} \longrightarrow \dots$ resulting from the iteration of the function f

$$x_0 \longrightarrow f(x_0) \longrightarrow f(f(x_0)) \longrightarrow \dots \ .$$

[MOUTHS TO FEED]

In **1798** the English Economist **Thomas R. Malthus** theorized that

"Population, when unchecked, increases in a geometrical ratio. Subsistence, increases only in an arithmetical ratio"

Fig. 4 - Thomas Robert Malthus (1766 - 1834)

	Function	Start	Orbit
Resources	$f(x) = x + c$	R	$R \longrightarrow R + c \longrightarrow R + 2c \longrightarrow \dots \longrightarrow R + nc \longrightarrow \dots$
Population	$f(x) = kx$	P	$P \longrightarrow kP \longrightarrow k^2P \longrightarrow \dots \longrightarrow k^nP \longrightarrow \dots$

Namely, people to be fed grow in an exponential way, while food increases only in a linear way. As a consequence, even on the increase of food production, there will always be a **no-return point**, in which the needs of the population exceed the amount of available food.

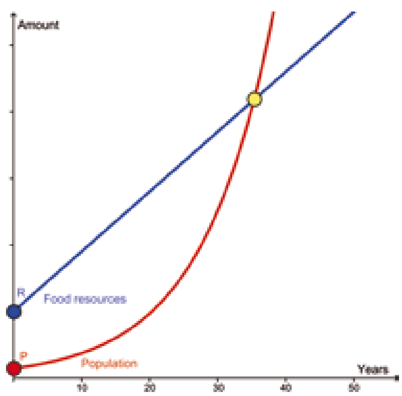


Fig. 5 - Food vs. People in Malthus' model

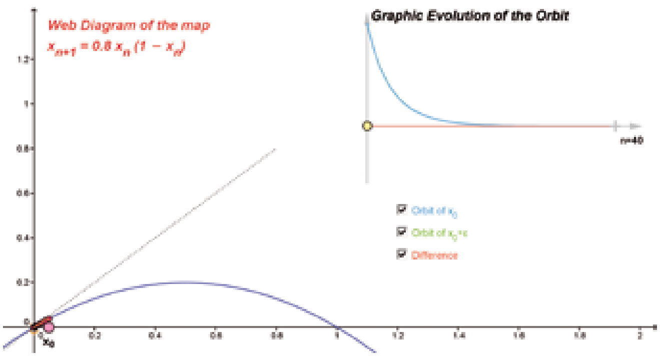
Fig. 6 - Pierre François Verhulst (1804 - 1849)

In **1839** the Belgian biologist **Pierre F. Verhulst** corrected Malthus' model by means of the **logistic map** $f(x) = kx(1 - x)$, which took into account the self-limitations in growth of a biological population, whose rate of reproduction is no longer constant over time (like in Malthus): it should be proportional not only to the existing population, but also to available resources.

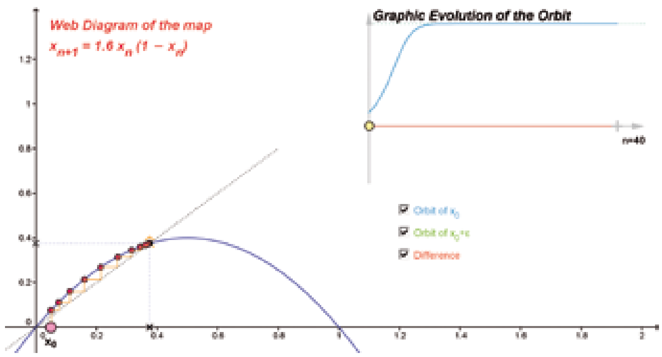
In **1976** the biologist **Robert May** ultimately proved the chaotic behavior of the logistic model for $k \approx 4$.

[C'EST LOGISTIQUE!]

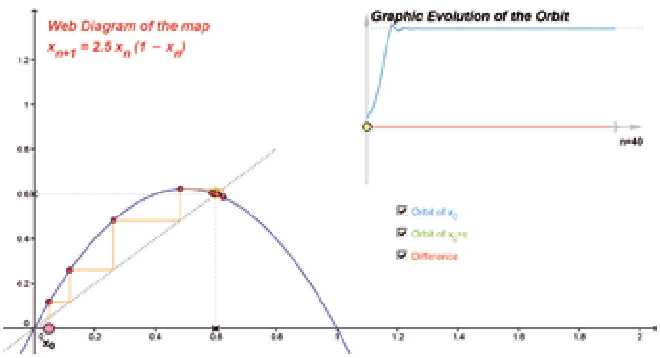
Given a starting value x_0, its orbit is calculated by **iterating** the function f. We use $0 \le x_0 \le 1$ and $0 \le k \le 4$. If we set $x_0 = 0.05$, we notice that the behavior of the logistic system deeply depends on the value of k. We draw both the **Web diagram** of the system and the graphic evolution of its orbit.



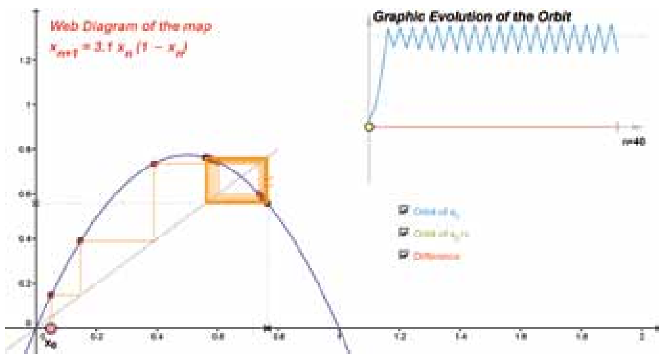
k = 0.8: the orbit falls to 0 (the population dies)



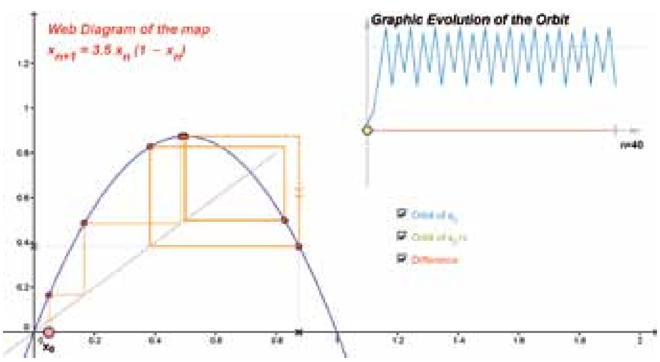
k = 1.6: the "sigma" shaped process rushes to a steady state



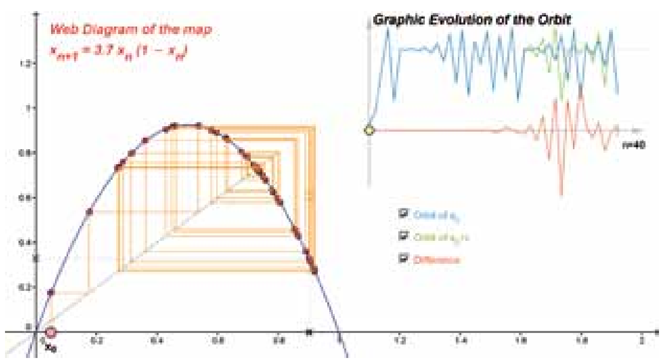
k = 2.5: the orbit stabilizes sharply again



k = 3.1: the process has period 2, around a limit value



k = 3.5: the quite regular orbit has now period 4



k = 3.7: the orbit of x_0 suddenly becomes chaotic.

There is more: if we consider the orbit of $x_0 + \varepsilon$ where $\varepsilon = 10^{-6}$, a value which is different but very close to x_0, we get an orbit which quickly differs from the first one, despite of being close to it at the beginning (see the red graph, now not flat like before). Here's the "**butterfly effect**".

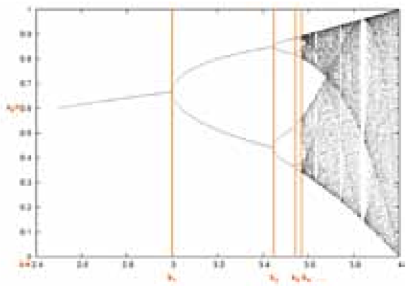
[THE FEIGENBAUM DIAGRAM]

The logistic map provides a simple and excellent example when examining the concept of Chaos. We can plot another chart, putting on the x-axis the parameter k and the starting point x_0 on y-axis, like the American mathematical physicist **Mitchell J. Feigen-**

Fig. 7 - Mitchell Jay Feigenbaum (1944 -)

baum did in 1974. For k < 3 the system is stationary; later on, a cascade of bifurcations starts and the period doubles each time. The ratio of corresponding intervals tends to the Feigenbaum constant δ:

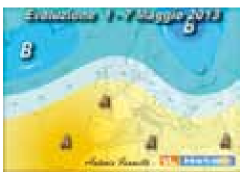
$$\frac{k_{n+1} - k_n}{k_{n+2} - k_{n+1}} \longrightarrow \delta = 4{,}669201609102990671853203820...$$



[APPLICATIONS]

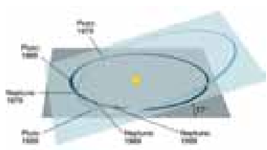
Geology: earthquakes cannot be exactly predicted

In Italy some scientists have been legally prosecuted and sentenced not because they were not able to predict 2009 earthquake (which would have been impossible), but because they did not consider the first shakes as a predicting sign, thus underestimating the danger. The earthquake destroyed L'Aquila city and caused 309 victims.



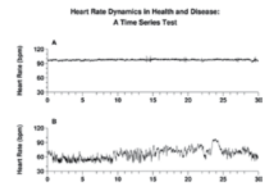
Meteorology: accurate long-term forecasts are impossible

Weather forecasts, although described by physics equations and processed by super computers, can be considered valid up to 80% within 48 hours, but then the percentage falls to 20% on the fourth day. This is due to the large number of factors that influence the weather in a given place, like small errors in the measurement of initial conditions or the rounding of values during calculations.



Astronomy: not all planet orbits are deterministic

Pluto does not orbit on the plane of the **Ecliptic** as well as the other planets of the Solar System do. Inclined of 17°, its orbit is so eccentric that it can be closer to the Sun than Neptune. In 1988 it was discovered that the orbit of Pluto is chaotic, as well.



Medicine: can we predict illness?

A healthy heart has a chaotic rhythm, while the rhythm of a suffering heart seems to be regular. Health is quite chaotic (especially in Italy!), whereas illness is linear.

[ECONOMICS: COULD THE CRISIS BE PREDICTED?]

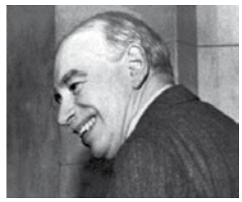


Fig. 8 - John Maynard Keynes (1883 - 1946)

Chaos acts, as we unfortunately know, even in Economics: **Stock Market crashes** are chaotic phenomena. But in the same way geologists can identify a seismic zone, economists should understand the conditions in which a traumatic event is more likely to happen. Perhaps, the financial crisis spreading since 2007 could have been prevented by paying more attention to "changes in initial conditions". Sometimes economists should consider that a little "snowball" can become an "avalanche". All in all, only one long-term effect is certainly predictable, like the British economist **John M. Keynes** said in 1923:

"*The long run is a misleading guide to current affairs. In the long run we are all dead*"

[GLOSSARY]

1. Deterministic = no random interference is involved, so that we always get the same output from a given initial stat

2. Difference Equation = equation that recursively defines a sequence starting from one or more initial terms

3. Iterating = applying repeatedly

4. Web diagram (or cobweb plot) = a useful way to view the evolution of a dynamical system based e.g. on the logistic map; it's not a trap for butterflies!

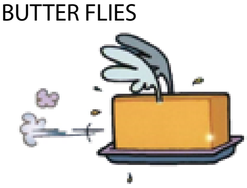
5. Ecliptic = the plane of Earth's orbit around the Sun

6. Stock Market crashes (trad. "crolli delle borse") = sudden fall of the value of capitals exchanged on the Market, like the "Black Tuesday" of Wall Street in October 1929 or the collapse of European Markets in May 2012

[THE FINAL PUN]

THE BUTTERFLIES

THE BUTTER FLIES



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