Prove di CLIL //

Logarithm: a simple tool, not a hard step

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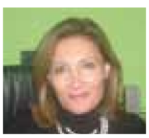
[NICOLA CHIRIANO]

Nicola Chiriano è docente di Matematica e Fisica al Liceo scientifico "Siciliani" di Catanzaro. Si occupa di didattica e ICT. È formatore in diversi corsi per docenti e studenti di vari ordini di scuola. Ha all'attivo varie collaborazioni con Ansas (e-tutor nei corsi Pon Tec) e Invalsi (piani di formazione Ocse-Pisa e SNV). Su A&B ha già proposto un percorso tra Musica e Matematica.



[CATERINA OLIVERIO]

Caterina Oliverio, laureata in Lingue e Letterature straniere presso l'Università di Bari, è docente di Lingua e Letteratura inglese nelle scuole superiori dal 1992. Dal 2000 insegna al Liceo scientifico "Siciliani" di Catanzaro, dove si occupa di English for Specific Purposes. Collabora a contratto con la Facoltà di Medicina e Chirurgia dell'Università "Magna Graecia" di Catanzaro.

[THE PROBLEM]

Try to find the underlying rule in the following sequences:

-3	-2	-1	0	1	2	3	4	5	6	7
1/27	1/9	1/3	1	3	9	27	81	243	729	2187

It is quite evident that the first line is obtained by **adding** 1 to the previous term, while the second line is obtained by **multiplying** by 3 the previous term. The first sequence is called **arithmetic progression**, the second one is called **geometric progression**. There is a cool relationship between the two lines: suppose we have to calculate

$$9 \times 243 \ .$$

By following a reversed pattern, we add the correspondent terms to 9 and 243 on the first line, which are 2 and 5:

$$2 + 5 = 7 \ .$$

The number below this result is actually the expected one:

$$2187 = 9 \times 243 \ .$$

Now, let's plot the two sequences of values on a graph:

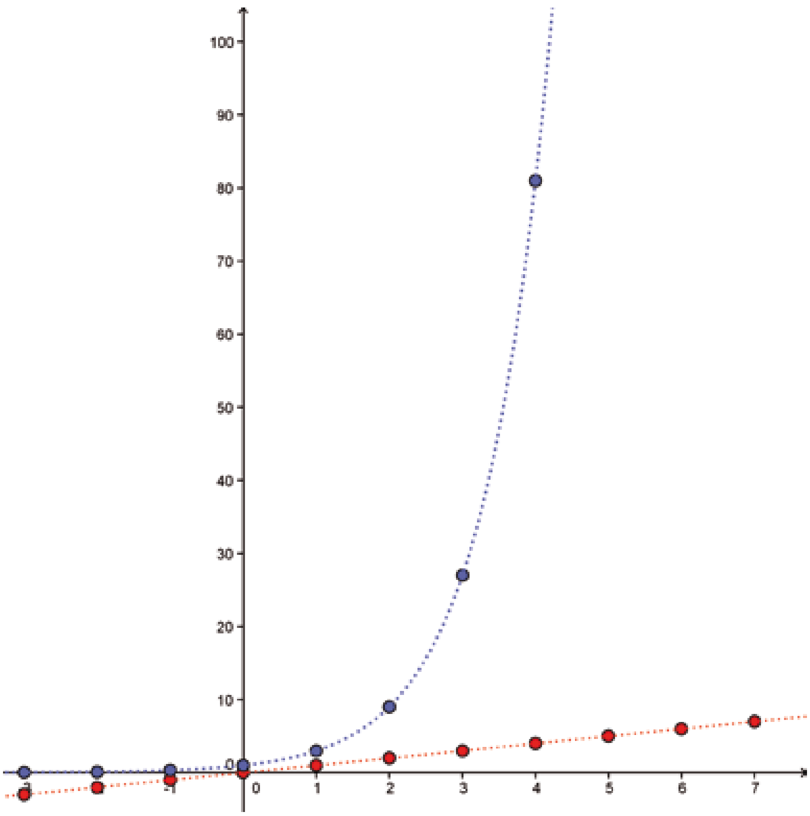


Figure 1

Since the terms of the first sequence (in red) have a constant difference, they lie on a straight line. The other values (in blue) bear a constant ratio one to the other and lie on an exponential function graph.

[THE SOLUTION]

Starting from this simple idea and developing extended detailed tables of this kind, thus using additions instead of multiplications, a new relationship among numbers was introduced in Maths by the Scottish **John Napier** (1614) and the English **Henry Briggs** (1624). This relationship was called **Logarithm** from the Greek terms λόγος (ratio, relation) and 'αριθμός (number).

So a Logarithm is able to turn products into sums and then ratios into differences: it was actually introduced as a calculation tool for simplification. In modern terms it comes to be defined as the exponent x which a given base a of a power must be elevated to in order to get a (positive) number b:

$$a^x = b \quad \longleftrightarrow \quad x = \log_a b$$

where

$$a > 0 \ \wedge \ a \neq 1 \ \wedge \ b > 0 \ .$$

By means of well-known rules of exponents, that is what logarithms are actually meant to be, we can easily demonstrate the four Logarithm rules:

1. $\log_a bc = \log_a b + \log_a c$ **Product Rule**: the logarithm (to base a) of a product is the sum of the logarithms of the factors.

2. $\log_a \frac{b}{c} = \log_a b - \log_a c$ **Quotient Rule**: the logarithm of a quotient is the logarithm of the numerator minus the logarithm of the denominator.

3. $\log_a b^c = c \cdot \log_a b$ **Power Rule**: (called "Elevator Rule" by the authors, since the exponent seems to "fall down"): the logarithm of a power is the exponent times the logarithm of the base.

4. $\log_c b = \frac{\log_a b}{\log_a c}$ **Change of base Rule**: or Golden Rule (by Euler): try to read it by yourself!

[EXERCISES]

Find an approximate value of the following logarithms, by using both [Log] and [ln] keys on your calculator:

$$\log_3 5, \quad \log_9 \sqrt{125}, \quad \log_3 \frac{1}{15} \quad .$$

[RESOLUTION]

• $\log_3 5 = \frac{\text{Log}5}{\text{Log}3} \approx \frac{0.69897}{0.47712} \approx 1.46497$

and also $\log_3 5 = \frac{\ln 5}{\ln 3} \approx \frac{1.609438}{1.098613} \approx 1.46497$

• $\log_9 \sqrt{125} = \log_9 5^{3/2} = \frac{3}{2} \log_9 5 = \frac{3}{2} \frac{\log_3 5}{\log_3 9} =$

$\approx \frac{3}{2} \frac{1.46497}{2} \approx \ldots$

• $\log_3 \frac{1}{15} = \log_3 1 - \log_3 (3 \cdot 5) = 0 - (\log_3 3 + \log_3 5) =$

$= -1 - \log_3 5 \approx \ldots$

[GLOSSARY]

1. $a^x = b \longleftrightarrow x = \log_a b$ **:** a raised to the power x is b if and only if x is the logarithm of b to base a

2. [Log] : common (or Brigg's) logarithm, to base 10

3. [ln] : natural logarithm, to base *e*, Napier's number

[THE FINAL PUN]

Could this be Napier's native home?