Prove di CLIL //

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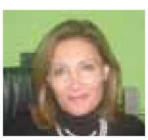
CONTINUA LA PROPOSTA DI ESPERIENZE IN LINGUA SECONDO IL CLIL (*CONTENT AND LANGUAGE INTEGRATED LEARNING*). GLI AUTORI SPERIMENTANO QUESTA METODOLOGIA IN COMPRESENZA NELLE ORE CURRICULARI, PROPONENDO AI PROPRI ALLIEVI DELLE UNITÀ DI APPRENDIMENTO INTERDISCIPLINARI DURANTE LE QUALI IL DOCENTE DI MATEMATICA PRESENTA I CONTENUTI E LA DOCENTE DI L2 (INGLESE) INTERAGISCE, MIRANDO ALL'AMPLIAMENTO LESSICALE E AL CONSOLIDAMENTO DELLE STRUTTURE LINGUISTICHE. LA VALUTAZIONE VIENE EFFETTUATA CONGIUNTAMENTE.
QUESTA PUNTATA È DEDICATA AI NUMERI COMPLESSI… NON COMPLICATI.
IN GIALLO SONO EVIDENZIATE *HOT WORDS* CHE VENGONO IN SEGUITO MEGLIO DEFINITE NEL GLOSSARIO.

[NICOLA CHIRIANO]
Nicola Chiriano è docente di Matematica e Fisica al Liceo scientifico "Siciliani" di Catanzaro. Si occupa di didattica e ICT. È formatore in diversi corsi per docenti e studenti di vari ordini di scuola. Ha all'attivo varie collaborazioni con Ansas (e-tutor nei corsi Pon Tec) e Invalsi (piani di formazione Ocse-Pisa e SNV). Ha proposto su A&B un percorso tra Musica e Matematica.



[CATERINA OLIVERIO]
Caterina Oliverio, laureata in Lingue e Letterature Straniere presso l'Università di Bari, è docente di Lingua e Letteratura Inglese nelle scuole superiori dal 1992. Dal 2000 insegna al Liceo scientifico "Siciliani" di Catanzaro, dove si occupa di English for Specific Purposes. Collabora a contratto con la Facoltà di Medicina e Chirurgia dell'Università "Magna Graecia" di Catanzaro.

Complex... but not complicated

[IMAGINE ALL THE... NUMBERS]

Consider this geometric problem:

"divide a segment of length 10 into two parts so that the rectangle having them as sides has area 40"

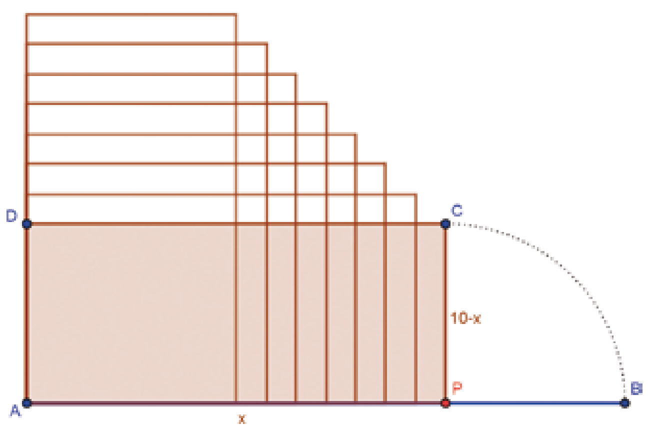


Figure 1

Since the maximum area of such a rectangle is 25, the problem has no solution in $\mathbb{R}$, that is to say, it has no geometric solution.

We can try an algebraic way to solve the problem: we label x and 10 - x the two parts the segment is splitted into, thus obtaining

$$x(10-x) = 0$$

namely

$$\mathbf{x^2 - 10x + 40 = 0} \ .$$

Note that if a and b are solutions to this equation, the following relations hold

$$a + b = 10 \ \text{ and } \ a \cdot b = 40 \ .$$

When **Girolamo Cardano** (*Ars Magna*, 1545) tried to solve the problem, despite of Tartaglia's attempts to discourage him, he obtained two "strange" quantities with an *imaginary part*:

$$5 + \sqrt{-15} \quad \text{and} \quad 5 - \sqrt{-15}$$

whose sum

$$(5 + \sqrt{-15}) + (5 - \sqrt{-15}) = 10$$

and product

$$(5 + \sqrt{-15}) \cdot (5 - \sqrt{-15}) = 25 - (-15) = 40$$

are actually what we were looking for.

Thanks to the works of **René Descartes** (1645) and **Leonard Euler** (1765), nowadays we write those numbers in the form

$$5 \pm i\sqrt{15}$$

where

$$i^2 := -1$$

is called the imaginary unit.

[PLOTTING IMAGINARY NUMBERS]

K.F. Gauss in his thesis (1799) and **J.R. Argand** (1806) developed a very simple method to "visualize" complex numbers, that is numbers in the form

$$z = a + ib$$

with a **real part** a = Re(z) and an **imaginary part** b = Im(z). Their idea was to represent z on the plane by the vector $\overline{OP}$ where P = (a,b).

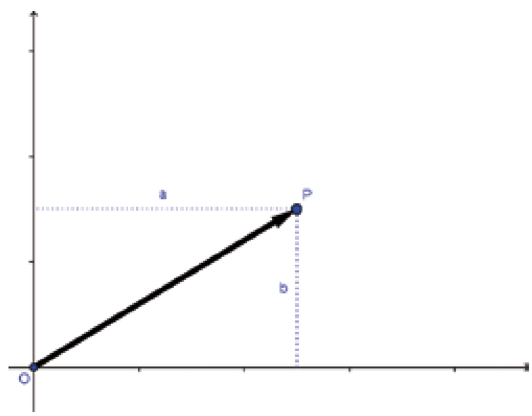


Figure 2

[(NOT SO) COMPLEX OPERATIONS]

We can treat complex numbers with the usual algebraic operations for real numbers. Therefore, if

$$z = a + ib \quad \text{and} \quad w = c + id$$

their sum is

$$z + w = (a + c) + i(b + d)$$

and their product

$$z \cdot w = (ac - bd) + i\,(ad + bc)\ .$$

We define **conjugate** of z the complex number

$$\bar{z} = a - ib$$

with the property

$$z \cdot \bar{z} = a^2 + b^2 \geq 0$$

and the **inverse** of z the complex number

$$\frac{1}{z} = \frac{\bar{z}}{z \cdot \bar{z}} = \frac{a}{a^2 + b^2} - i\frac{b}{a^2 + b^2}$$

useful to perform a division in the guise of a product:

$$\frac{w}{z} = w \cdot \frac{1}{z}\ .$$

[POLAR FORM]

By using a little Trigonometry,

$$a = \rho\cos\theta, \quad b = \rho\sin\theta$$

where

• $\rho = |z| = \sqrt{a^2 + b^2}$ is the length of the vector z: it's called **modulus of z**

• θ is the angle formed by z and the x axis: it's called the **argument of z**

So z can be written in the equivalent "polar" or trigonometric form:

$$z = \rho(\cos\theta + i\sin\theta)\ .$$

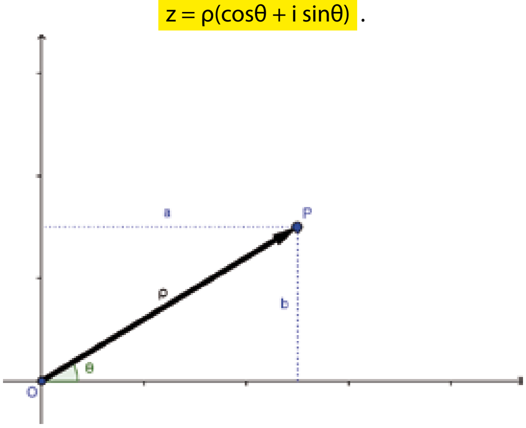


Figure 3

[EULER'S FORMULA]

In 1743 Euler discovered an outermost relationship between three basic functions: exponential in base e, sine and cosine:

$$e^{i\pi} = \cos x + i\sin x\ .$$

With the value, we obtain Euler's "the formula",

$$e^{i\pi} + 1 = 0$$

a masterpiece known as "the most beautiful formula" of Mathematics.

[EXERCISE]

Let's try to write the complex number

$$z = 1 + i$$

into its polar and exponential forms. Since $\rho = \sqrt{2}$ and $\theta = \pi/4$, we get

$$z = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = \sqrt{2}e^{i\,\pi/4}$$

Little check:

$$\sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = \sqrt{2}(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}) = 1 + i\ .$$

[GLOSSARY]

1. i^2: = -1 : i squared is by definition equal to negative one

2. $z = \rho(\cos\theta + i\sin\theta)$: zed is rho times (= multiplied by) cosine of theta plus sine of theta

3. $e^{i\pi} + 1 = 0$: e (Napier's number) elevated (or "to the power") i times pi plus one is zero

[AND FINALLY...]

Explain why the following strip can make people smile: